

GPU-accelerated large-scale excited-state simulation based on divide-and-conquer time-dependent density-functional tight-binding

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Introduction

- Waseda University
PhD, Department of Chemistry and Biochemistry, Nakai group (March, 2015)
- April 2015- March 2016
Research Associate, Waseda University
- April 2016- March 2018
Assistant Professor, Waseda University
- April 2018- Present
Lecturer, Waseda University

Linear-scaling open-shell and excited-state theory

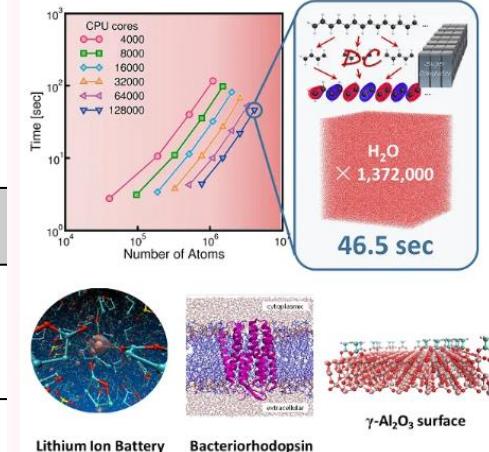
Contents

- DC-DFTB-MD
 - Divide-and-conquer density-functional tight-binding molecular dynamics
 - Ground state, massively parallel
- DC-TDDFTB-MD
 - Divide-and-conquer time-dependent density-functional tight-binding molecular dynamics
 - Excited state
- Acceleration of DC-TDDFTB-MD with GPU

Development of DC-DFTB-MD program: DC-DFTB-MD

- MD and geometry optimization for large systems
- Written in Fortran 90/95
- K computer, Fujitsu FX10, PC cluster (Linux), ...

Academic free



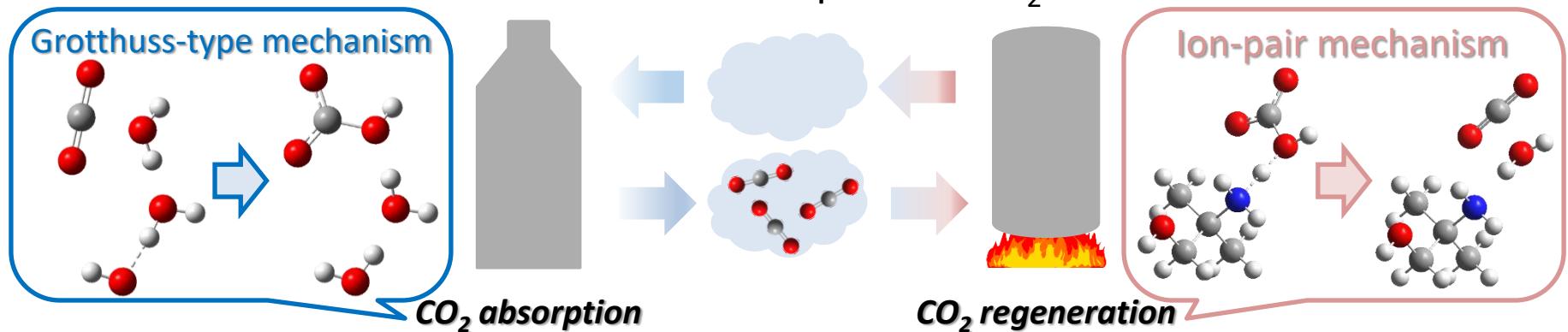
Main features

Energy and Gradient (Cluster and PBC models)	DFTB1/2/3, Spin-polarized DFTB
Linear-scaling method	Divide-and-conquer (DC)
Dispersion correction	Slater-Kirkwood, Lennard-Jones, DFT-D2, DFT-D3
SCC convergence	Broyden, Simple mixing, Anderson, DIIS
MD ensemble	<i>NVE, NVT, NPH, NPT</i>
Thermostat and [Barostat]	Velocity scaling, Nosé-Hoover chain, Berendsen, Andersen, Langevin, [Berendsen]
MD option	RATTLE constraint, Soft potential, Simulated annealing, Lagrange interpolation of initial guess charges
Geometry optimization	BFGS, Steepest descent, Conjugate gradient

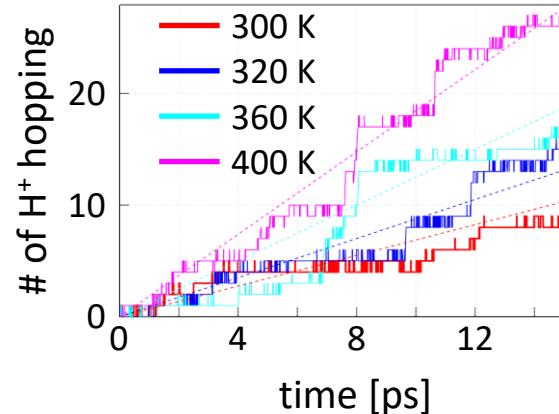
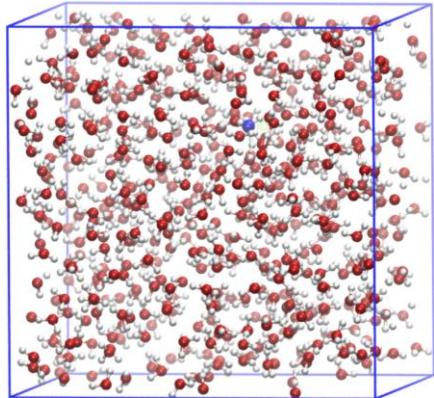
Target: Chemical reaction dynamics of large Systems

Our recent illustrative applications

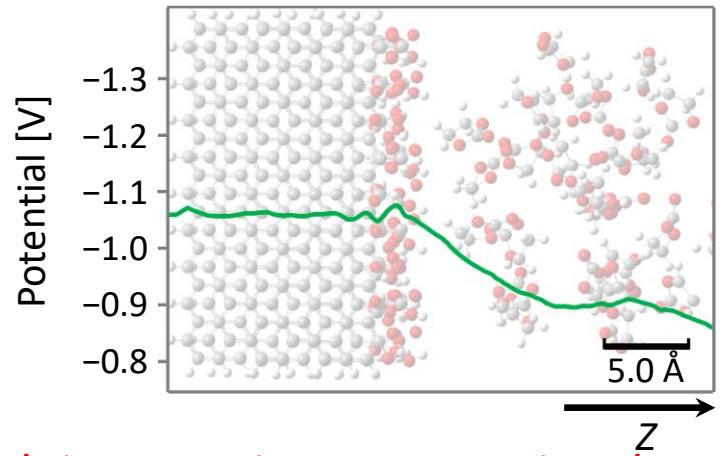
Chemical absorption of CO₂



Proton diffusion in bulk water



Lithium ion devices

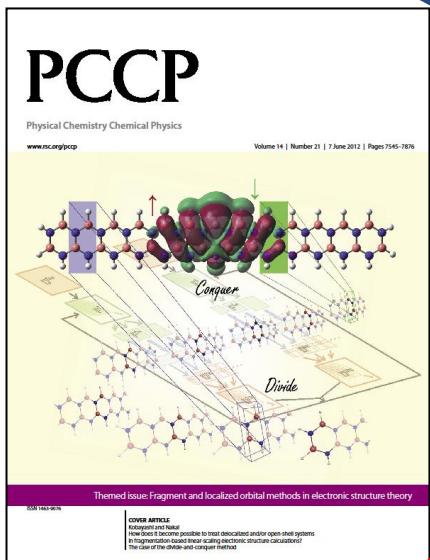


Quantum mechanical treatment of whole system with inexpensive computational cost

Our three pillars for chemical reaction dynamics

DC

**Divide-
and-
Conquer
 $O(N)$**



DFTB

**Density-
Functional
Tight-
Binding
SEQM-MD**



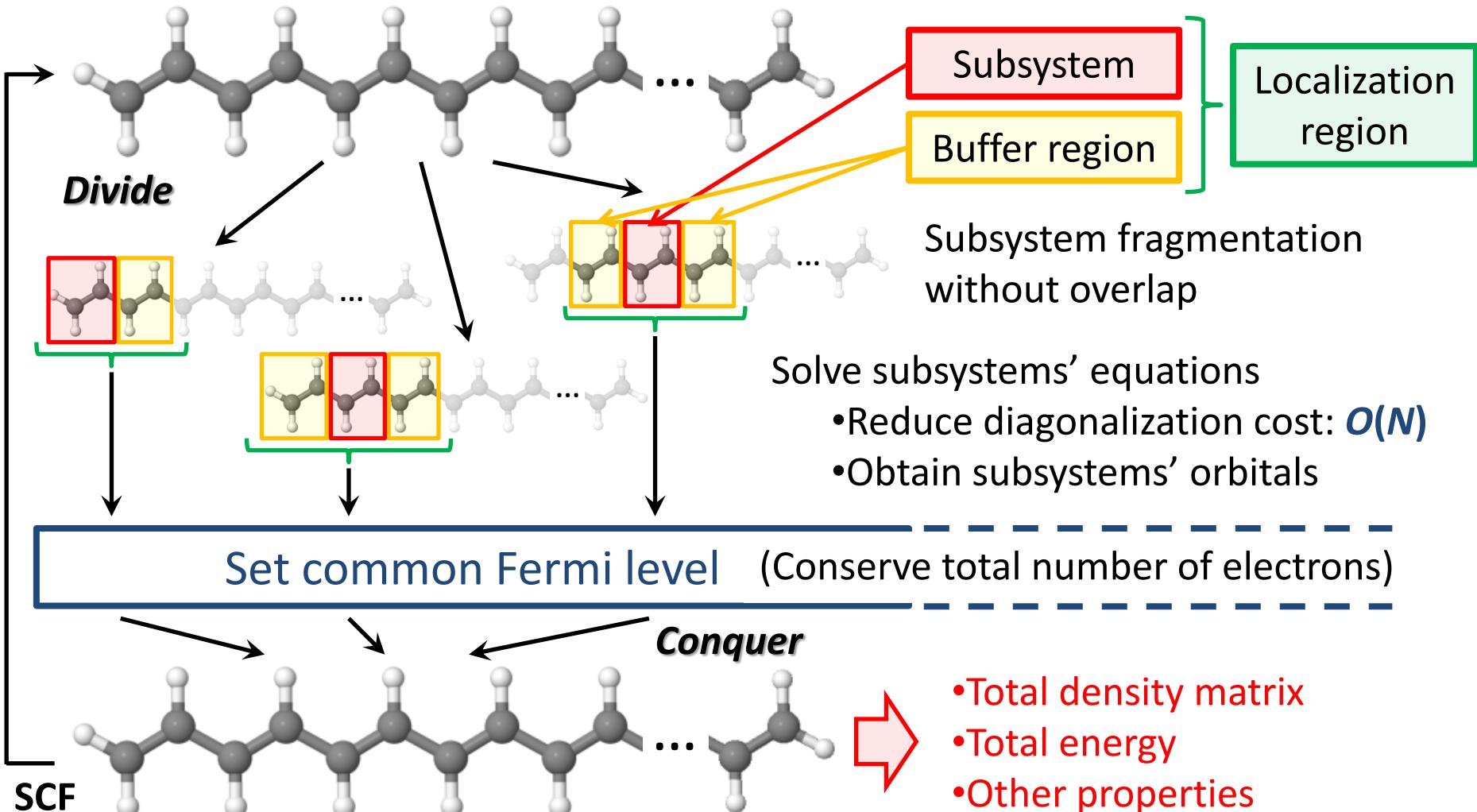
K computer

***Massively parallel
DC-DFTB-MD
computations***

- $O(N)$: Linear-scaling
- SEQ-MD: Semi-Empirical Quantum Mechanics-Molecular Dynamics

Divide-and-Conquer (DC) method

One of fragmentation techniques to achieve linear-scaling^[1,2]

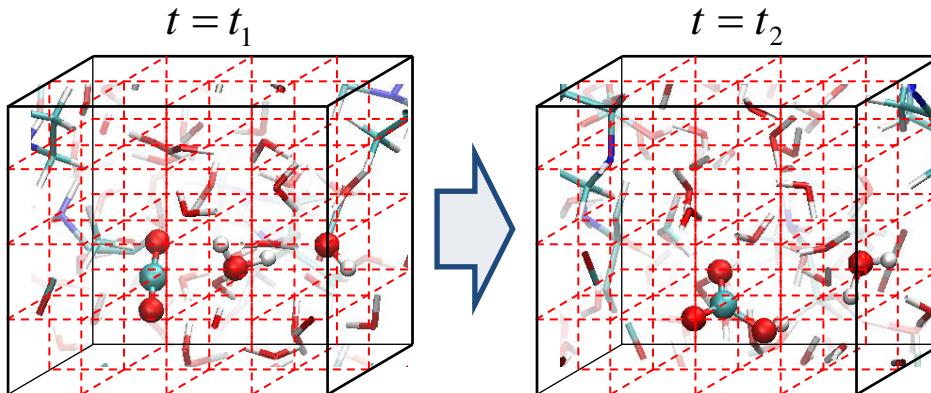


[1] M. Kobayashi, H. Nakai, in *Linear-Scaling Techniques in Computational Chemistry and Physics*, (2011), pp. 97–127.

[2] W. Yang, T.-S. Lee, *J. Chem. Phys.* **103**, 5674 (1995).

Advantages of DC method in QM-MD simulations

- Automatized fragmentation of subsystems at each time step



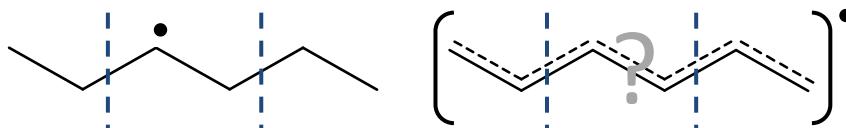
- Fragmentation due to locality of density matrix
- Accuracy and cost control by changing buffer size
- No need of chemical intuition

Applicable to dynamical bond formation and cleavage

- Unnecessary to predefine electron & spin numbers of subsystems
(Determine at each SCF step)

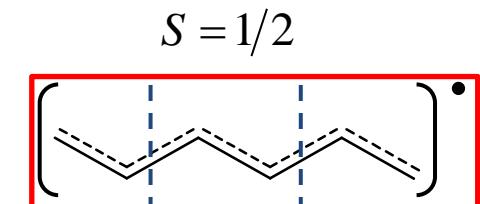
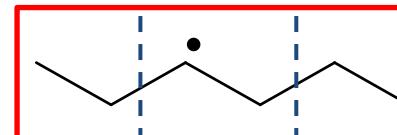
Conventional fragmentation method

$$S_1 = 0 \quad S_2 = 1/2 \quad S_3 = 0 \quad S_1 = ? \quad S_2 = ? \quad S_3 = ?$$



DC method

$$S = 1/2$$



Applicable to electron and spin delocalized systems

Density-Functional Tight-Binding (DFTB) method

SEQM derived from density functional theory (DFT)^[1,2]

$$E_{\text{DFTB}} = \sum_{\mu\nu}^{\text{AO}} D_{\mu\nu} H_{\mu\nu}^0 + \sum_{A>B}^{\text{atom}} E_{AB}^{\text{rep}} + \frac{1}{2} \sum_{AB}^{\text{atom}} \gamma_{AB} \Delta q_A \Delta q_B + \frac{1}{3} \sum_{AB}^{\text{atom}} \Gamma_{AB} \Delta q_A^2 \Delta q_B$$

Evaluate using parameters determined by DFT calculations

①: Charge independent term

- Precomputed Hamiltonian and overlap matrices
- Two-center approximation

→ No integral evaluation at runtime

②: Short-ranged two-body repulsive term

- Core-core repulsion
- DFT double-counting contribution

→ Key ingredient for reasonable accuracy and transferability

③, ④: Charge dependent term

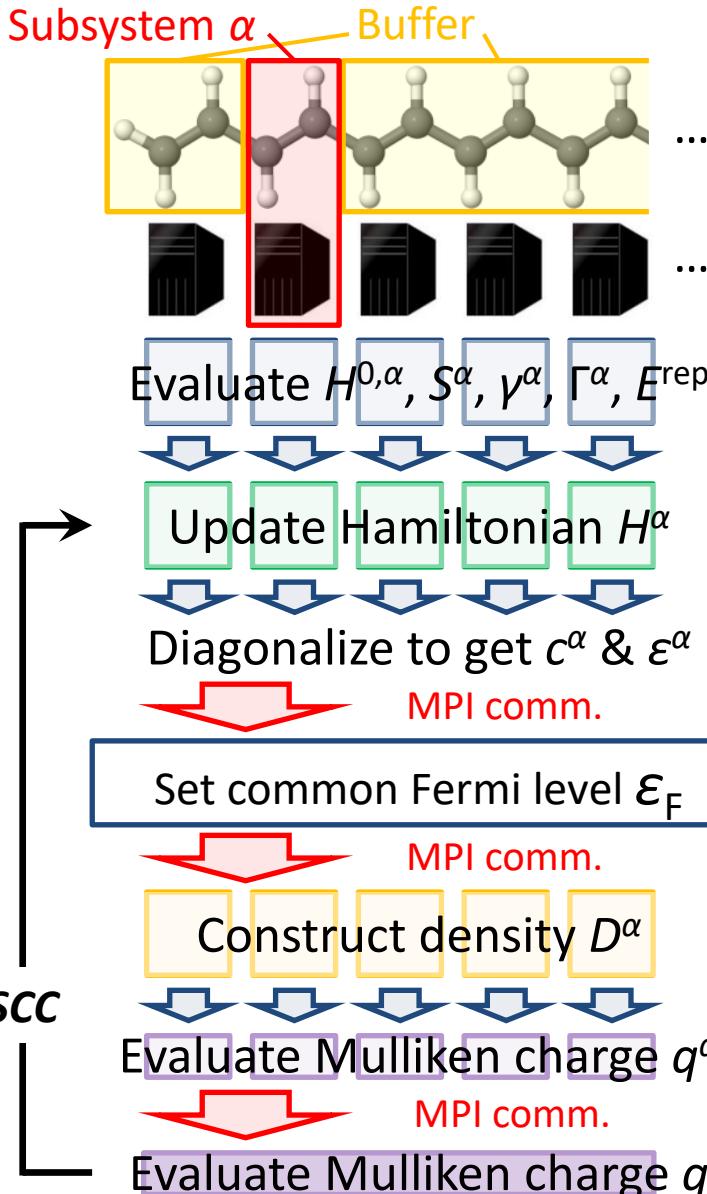
- Δq_A is determined self-consistently (SCC)
- Monopole approximation

→ Proper description of chemical bonds in MD with small cost

[1] M. Elstner, G. Seifert, *Phil. Trans. R. Soc. A* **372**, 20120483 (2014).

[2] M. Gaus, Q. Cui, M. Elstner, *WIREs Comput. Mol. Sci.* **4**, 49 (2014).

Parallel procedure of DC-DFTB energy calculation



- Hybrid of MPI/OpenMP

- MPI: Assign subsystems to processes
- OpenMP: Operations for localization regions

$$H_{\mu\nu} = H_{\mu\nu}^0 + S_{\mu\nu} \sum_{\text{atom}} \frac{1}{2} (\gamma_{AC} + \gamma_{BC}) \Delta q_C$$

$$+ S_{\mu\nu} \sum_{\text{atom}} \left(\frac{1}{3} (\Delta q_A \Gamma_{AC} + \Delta q_B \Gamma_{BC}) + \frac{\Delta q_C}{6} (\Gamma_{CA} + \Gamma_{CB}) \right) \Delta q_C$$

→ Solve equations: $\sum_v c_{vi}^\alpha (H_{\mu\nu}^\alpha - \epsilon_i^\alpha S_{\mu\nu}^\alpha) = 0$

Constraint of total number of electrons n_e :

$$n_e = 2 \sum_{\alpha}^{\text{subsystemMO}(\alpha)} \sum_i^{\text{MO}(\alpha)} f_\beta (\epsilon_F - \epsilon_i^\alpha) \sum_{\mu\nu} p_{\mu\nu}^\alpha c_{\mu i}^\alpha c_{\nu i}^* S_{\nu\mu}^\alpha$$

$$D_{\mu\nu}^\alpha = 2 \sum_i^{\text{MO}(\alpha)} f_\beta (\epsilon_F - \epsilon_i^\alpha) c_{\mu i}^\alpha c_{\nu i}^*$$

$$q_A^\alpha = \sum_{\mu \in A} \sum_{\nu \in L(\alpha)} D_{\mu\nu}^\alpha S_{\nu\mu}^\alpha$$

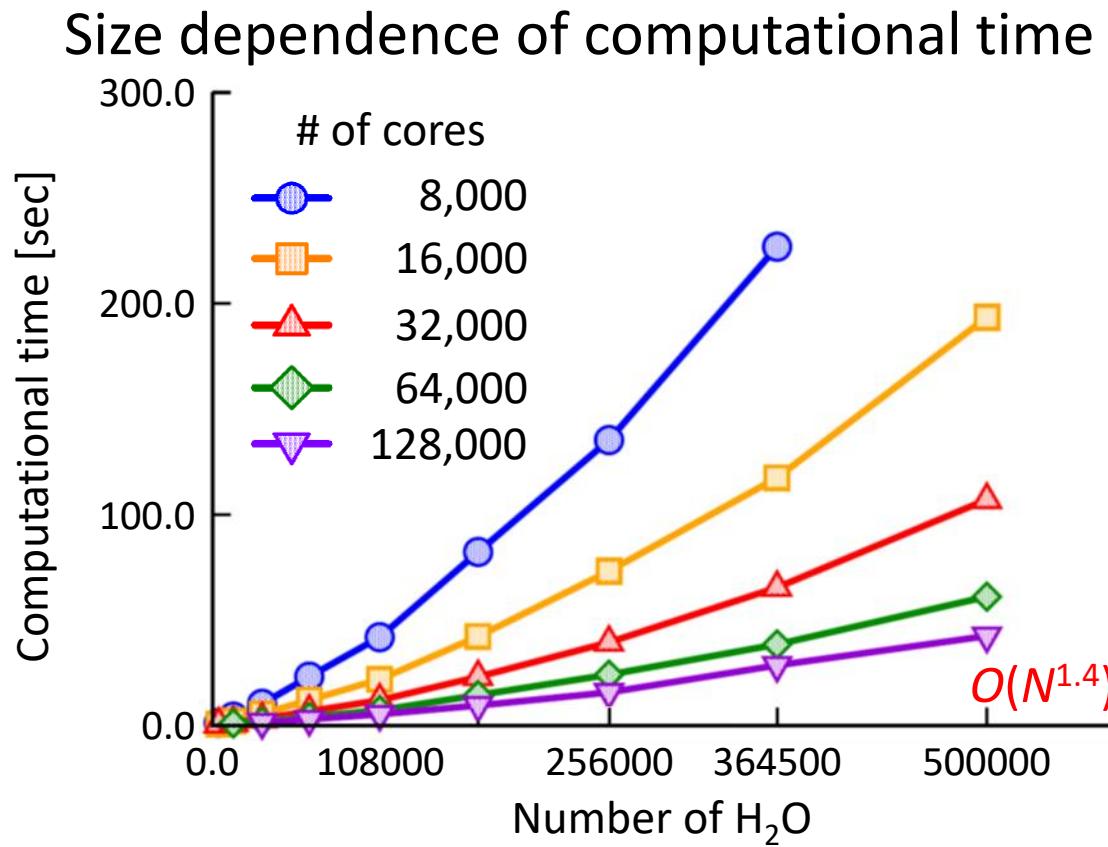
➤ $f_\beta(x)$: Fermi function

$$f_\beta(x) = \frac{1}{1 + \exp(-\beta x)}$$

➤ p^α : partition matrix

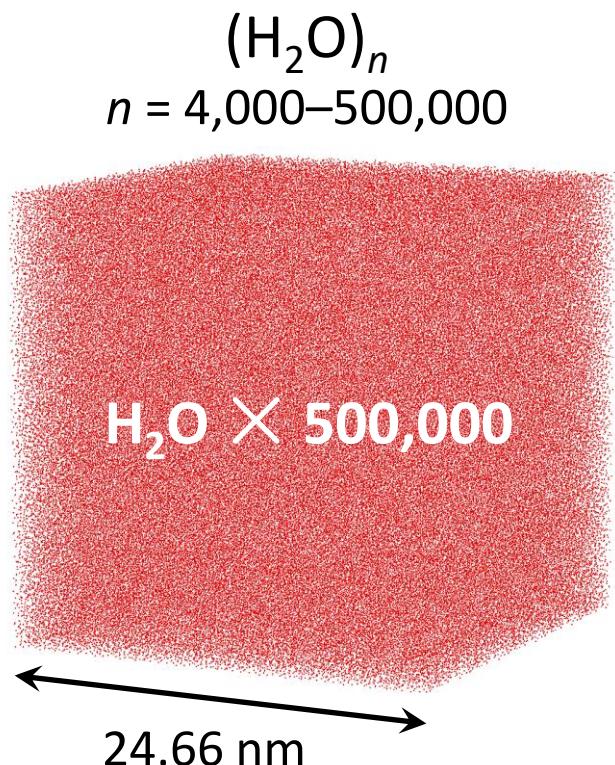
→ SCC Convergence check: $\max |q - q_{\text{old}}| < \delta$

Parallel performance of DC-DFTB energy calculation



Computational time of $(\text{H}_2\text{O})_{500,000}$

# of cores	16,000	32,000	64,000	128,000
Time [sec]	193.4	107.0	61.1	46.5
Speed-up	16000.0	28908.7	50673.9	72935.4



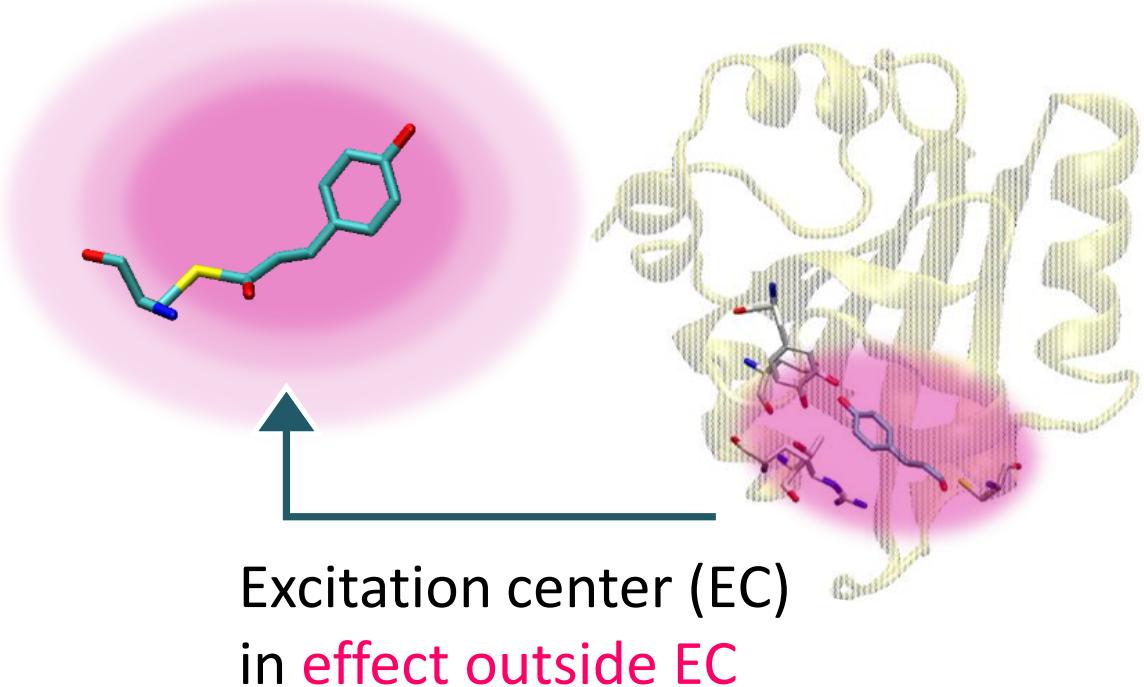
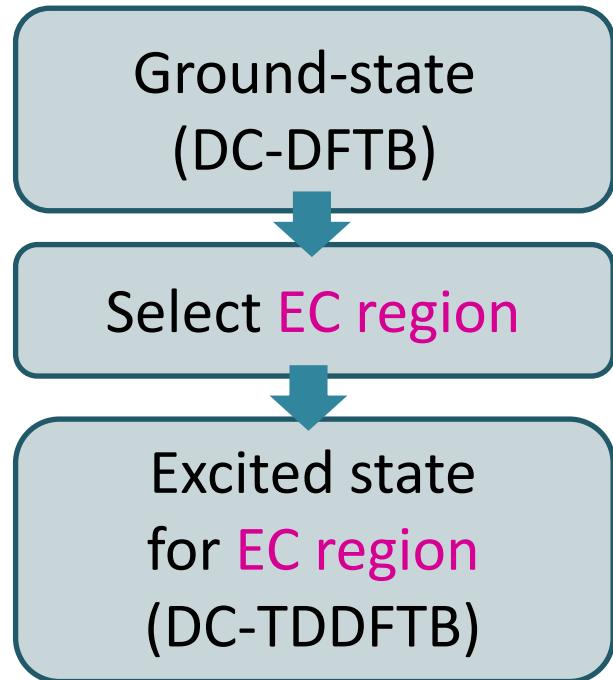
- Density: 1.0 g/cm³
- DC-DFTB2
- Subsystem: 1 H_2O
- Buffer radius : 6 Å
- K computer

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DC-based excited-state theory

DC-based time-dependent density-functional tight-binding theory



Casida's formalism

$$\begin{pmatrix} \mathbf{A}^s & \mathbf{B}^s \\ \mathbf{B}^{s*} & \mathbf{A}^{s*} \end{pmatrix} \begin{pmatrix} \mathbf{X}^s \\ \mathbf{Y}^s \end{pmatrix} = \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{X}^s \\ \mathbf{Y}^s \end{pmatrix}$$

[1] T. Yoshikawa, M. Kobayashi, A. Fujii, and H. Nakai, *J. Phys. Chem. B*, **117**, 5565 (2013).

[2] N. Komoto, T. Yoshikawa, J. Ono, Y. Nishimura, and H. Nakai, *J. Chem. Theor. Comput.*, **15**, 1719 (2019).

Time-dependent DFT(B) energy/energy gradient

- Excitation energy: linear-response theory
 - Hermit eigenvalue problem: Davidson algorithm

$$(\mathbf{A} - \mathbf{B})^{1/2} (\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})^{1/2} \mathbf{c}_I = \omega_I^2 \mathbf{c}_I$$

$$A_{ia,jb} = \delta_{ij} \delta_{ab} (\varepsilon_a - \varepsilon_i) + K_{ia,jb} \quad B_{ia,jb} = K_{ia,bj}$$

ε : orbital energy i,j : occupied orbital a,b : unoccupied orbital

- Excitation energy gradient

$$\frac{\partial \omega}{\partial R_{\alpha x}} = \sum_{\mu\nu} \frac{\partial H_{\mu\nu}^0}{\partial R_{\alpha x}} \Delta D_{\mu\nu} + \frac{1}{2} \sum_{\mu\nu\lambda\sigma} \frac{\partial K_{\mu\nu\lambda\sigma}}{\partial R_{\alpha x}} \Delta \Gamma_{\mu\nu\lambda\sigma} - \sum_{\mu\nu} \frac{\partial S_{\mu\nu}}{\partial R_{\alpha x}} \Delta W_{\mu\nu}$$

- Z-vector method

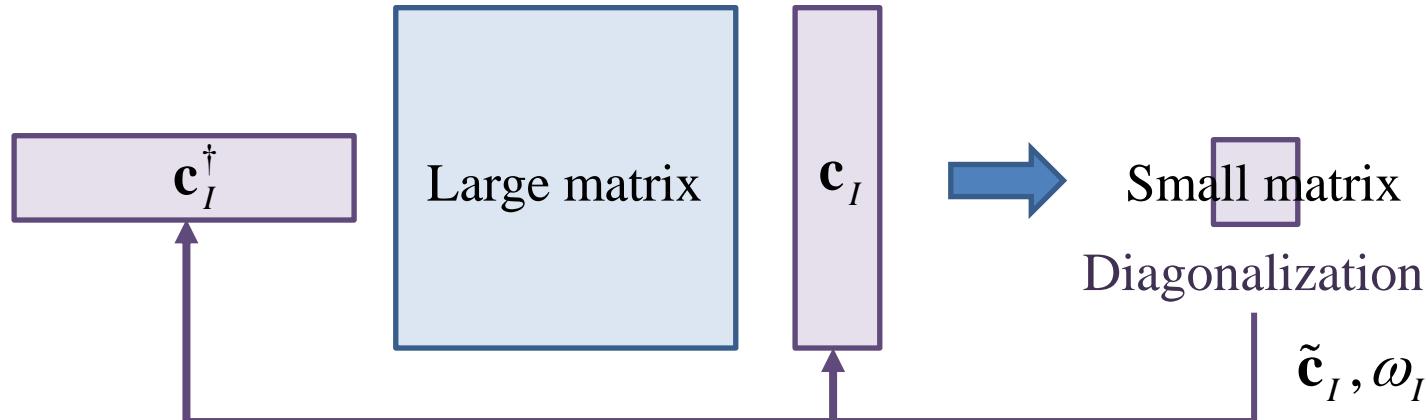
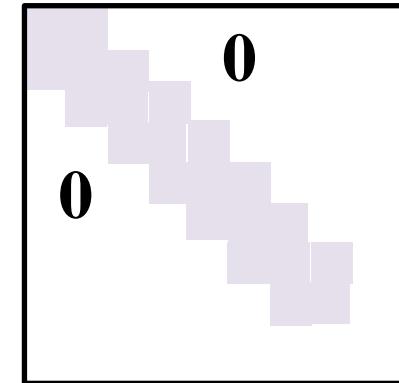
- Evaluate one-body, energy-weighted, and two-body density matrices
- Avoid the explicit evaluation of MO coefficient derivative for $3 \times N_{\text{atom}}$

TDDFTB energy: Davidson algorithm

- Diagonalization for sparse matrix
 - Hermit eigenvalue problem

$$\Omega \mathbf{c}_I = \omega_I^2 \mathbf{c}_I$$

$$\Omega = (\mathbf{A} - \mathbf{B})^{1/2} (\mathbf{A} + \mathbf{B}) (\mathbf{A} - \mathbf{B})^{1/2}$$



TDDFTB energy: Davidson algorithm

- Coupling matrix K (for example DFTB2)

$$K_{ia,jb} = \sum_{\alpha,\beta}^{Atom} q_{\alpha}^{ia} \gamma_{\alpha\beta} q_{\beta}^{jb} \quad \begin{matrix} \mu, v: \text{AO} \\ \alpha, \beta: \text{Atom} \end{matrix}$$

- Mulliken atomic transition charges

$$q_{\alpha}^{ia} = \frac{1}{2} \sum_{\mu \in \alpha} \sum_{\nu} \left(C_{\mu i} S_{\mu\nu} C_{\nu\alpha} + C_{\nu i} S_{\nu\mu} C_{\mu\alpha} \right) \quad \begin{matrix} \mathbf{C}: \text{MO coefficient} \\ \mathbf{S}: \text{overlap matrix} \end{matrix}$$

- Construction of $\Omega = (\mathbf{A} - \mathbf{B})^{1/2} (\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})^{1/2}$
 - Standard: AO-MO transformation for each Davidson iteration

$$\Omega_{ia,I} = \omega_{ia}^2 F_{ia} + 4 \sum_{jb} \sum_{\mu\nu\lambda\sigma} \sqrt{\omega_{ia}} C_{\mu i} C_{\nu\alpha} K_{\mu\nu, \lambda\sigma} \sqrt{\omega_{jb}} C_{\lambda j} C_{\sigma b} C_{jb,I}$$

- In-core: store q_{α}^{ia} in memory

$$\Omega_{ia,I} = \omega_{ia}^2 F_{ia} + 4 \sum_A q_A^{ia} \sqrt{\omega_{ia}} \left(\sum_B \gamma_{AB} \left(\sum_{jb} \sqrt{\omega_{jb}} q_A^{jb} F_{jb} \right) \right)$$

Algorithm of (DC-)TDDFTB

Step 1: Set initial trial vector

Sort in ascending order for difference of orbital energies

$$\omega_{ia} = \varepsilon_a - \varepsilon_i \quad \varepsilon : \text{orbital energy} \quad i,j: \text{occupied orbital} \quad a,b: \text{unoccupied orbital}$$

Step 2: Construct hermit matrix Ω

Standard

Step 2-1: MO-AO transformation
for trail vector

Step 2-2: Construct AO based Ω

Step 2-3: AO-MO transformation Ω

In-core

Step 2-1: Construct Mulliken
atomic transition charges

Step 2-1: Construct Ω directly
from trial vector

Step 3: Diagonalize hermit matrix Ω

SCF

Step 4: Update trial vector

SCF

TDDFTB energy gradient: Z-vector method

- Lagrangian of excitation energy in TDDFTB

$$\mathcal{L} = \langle X + Y | (A + B) | X + Y \rangle + \langle X - Y | (A - B) | X - Y \rangle - \omega \left\{ \langle X + Y | X - Y \rangle - 1 \right\} + \sum_{ia} Z_{ia} H_{ia} - \sum_{pq, p \leq q} \Delta W_{pq}^{TD} (S_{pq} - \delta_{pq})$$

|

- Z-Vector equation

$$\frac{\partial \mathcal{L}}{\partial C_{\mu p}} = 0 \longrightarrow \sum_{jb} (A + B)_{ia, jb} Z_{jb} = -R_{ia}$$

Stationary condition

Linear equations

$$R_{ia} = \sum_b \left\{ (X + Y)_{ib} H_{ab}^+ [(\mathbf{X} + \mathbf{Y})] \right\} + H_{ia}^+ [\mathbf{T}] \quad H_{pq}^+ [\mathbf{V}] = \sum_{ru} K_{pq, ru} V_{ru}$$

$$- \sum_j \left\{ (X + Y)_{ja} H_{ji}^+ [(\mathbf{X} + \mathbf{Y})] \right\}.$$

- One-body, two-body, and energy weighted DM by Z-vector

$$\frac{\partial \omega}{\partial R_{\alpha x}} = \sum_{\mu\nu} \frac{\partial H_{\mu\nu}^0}{\partial R_{\alpha x}} \Delta D_{\mu\nu} + \frac{1}{2} \sum_{\mu\nu\lambda\sigma} \frac{\partial K_{\mu\nu\lambda\sigma}}{\partial R_{\alpha x}} \Delta \Gamma_{\mu\nu\lambda\sigma} - \sum_{\mu\nu} \frac{\partial S_{\mu\nu}}{\partial R_{\alpha x}} \Delta W_{\mu\nu}$$

TDDFTB energy gradient: Z-vector method

- Preconditioned conjugate gradient (PCG) method
 - Solve iteratively symmetric and positive linear equations

Step 1: Construct Z-vector

Step 1-1: Set initial Z vector (Jaccobi preconditioner) $Z_{ia}^0 = \varepsilon_a - \varepsilon_i$

Step 1-2: Construct residual vector

$$r_{ia}^{k+1} = r_{ia}^k - \alpha^k \sum_{jb} (A + B)_{ia,jb} d_{ia}^k$$

SCF

Step 1-3: Update Z vector

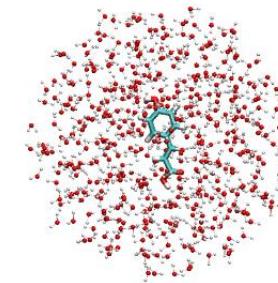
$$Z_{ia}^{k+1} = Z_{ia}^k + \alpha^k d_{ia}^k$$

Step 2: Construct density matrix

Step 3: Evaluate gradient

Performance of (DC-)TDDFTB method

- Wall-clock time [second]
 - Molecular: *p*-cumaric acid + 228H₂O
 - Method: TDDFTB2/3ob



Energy

	TDDFTB	DC-TDDFTB
Step 1	0.367	0.137
Step 2	1.943	0.293
Step 3	0.000	0.000
Step 4	0.127	0.037

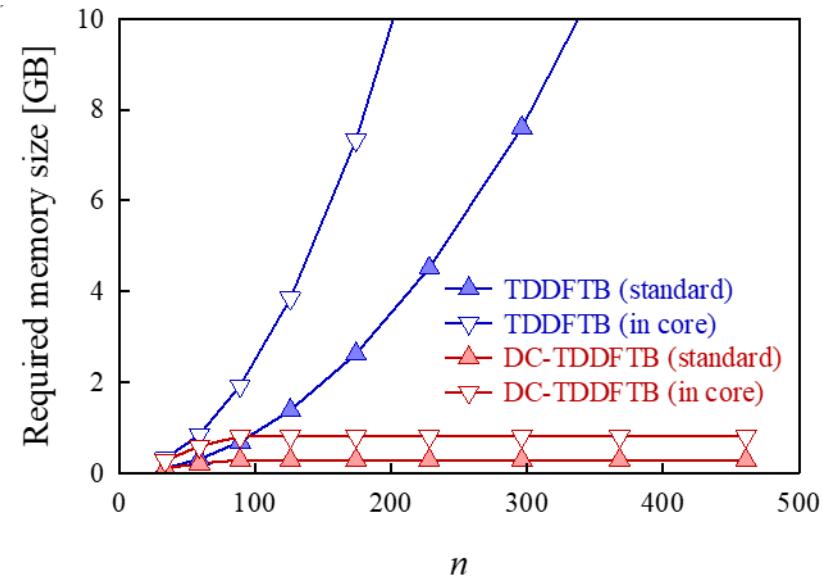
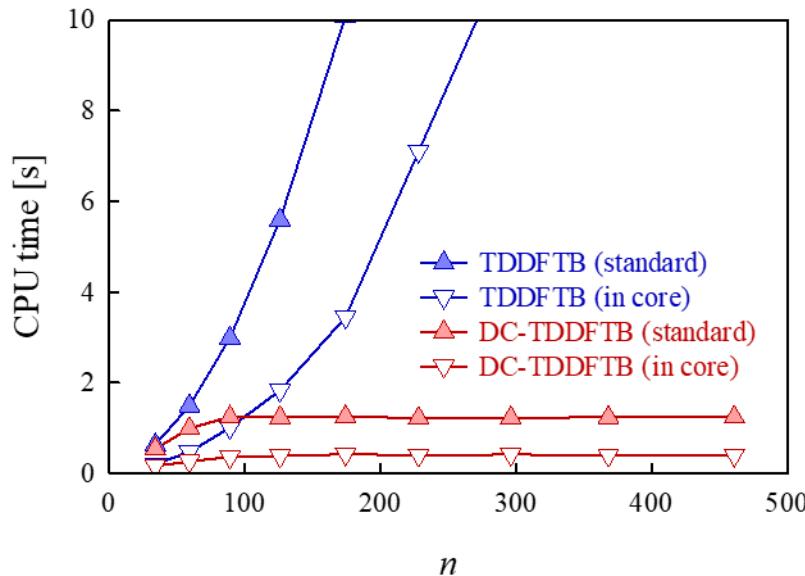
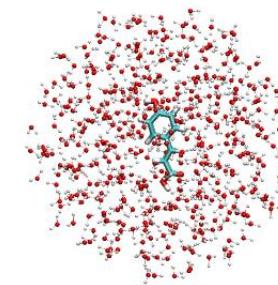
Energy gradient

	TDDFTB	DC-TDDFTB
Step 1	19.565	4.222
Step 2	0.200	0.066
Step 3	0.240	0.091

✓ DC calculations reduce the computational cost,
especially for largest computational step in each algorithm.

System-size dependency: total TDDFT calculation

- CPU time and required memory
 - Molecular: *p*-cumaric acid + $n\text{H}_2\text{O}$
 - Method: TDDFTB2/3ob

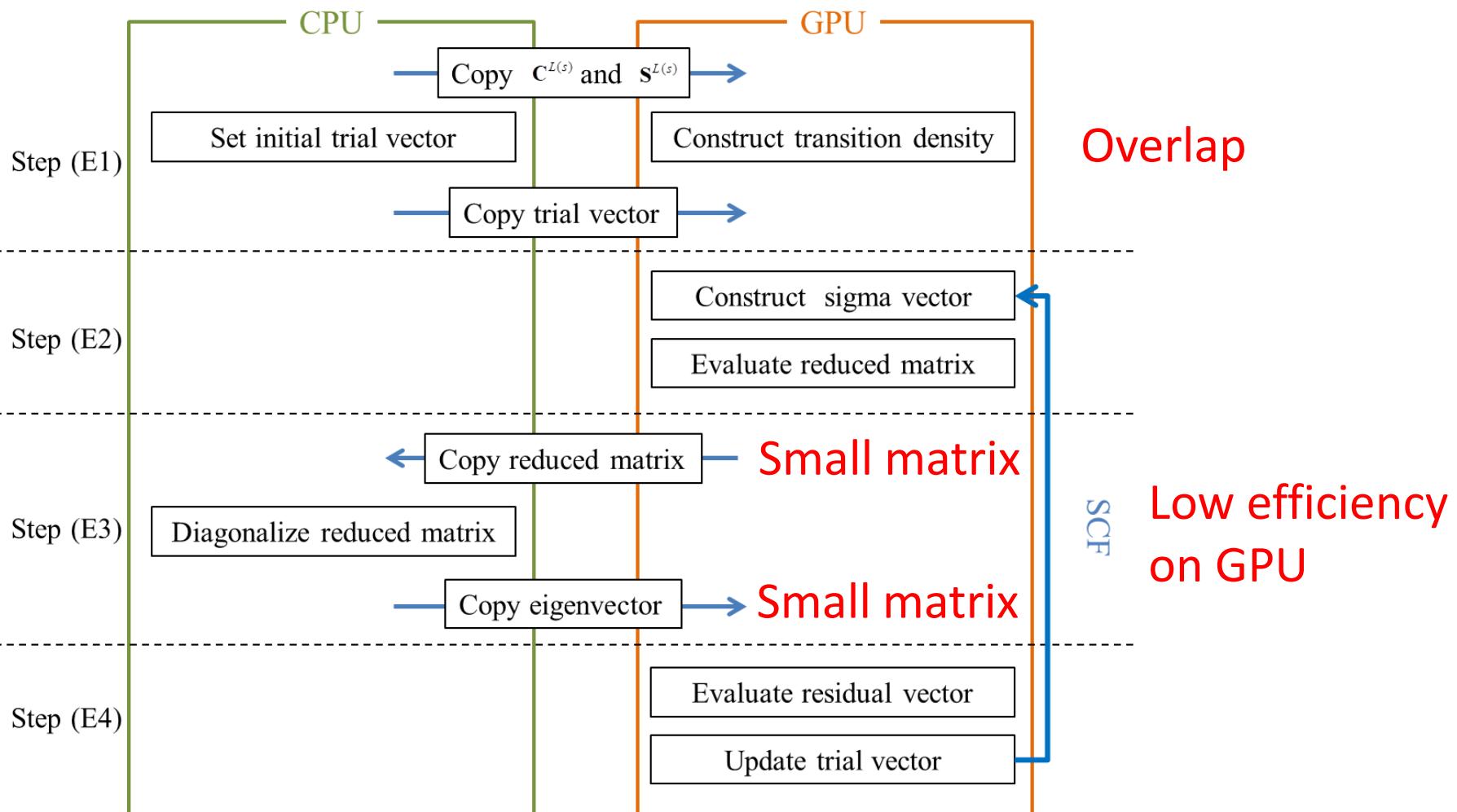


- ✓ In core algorithm reduce the wall times by large memory region
- ✓ DC calculations reduce the CPU time and required memory

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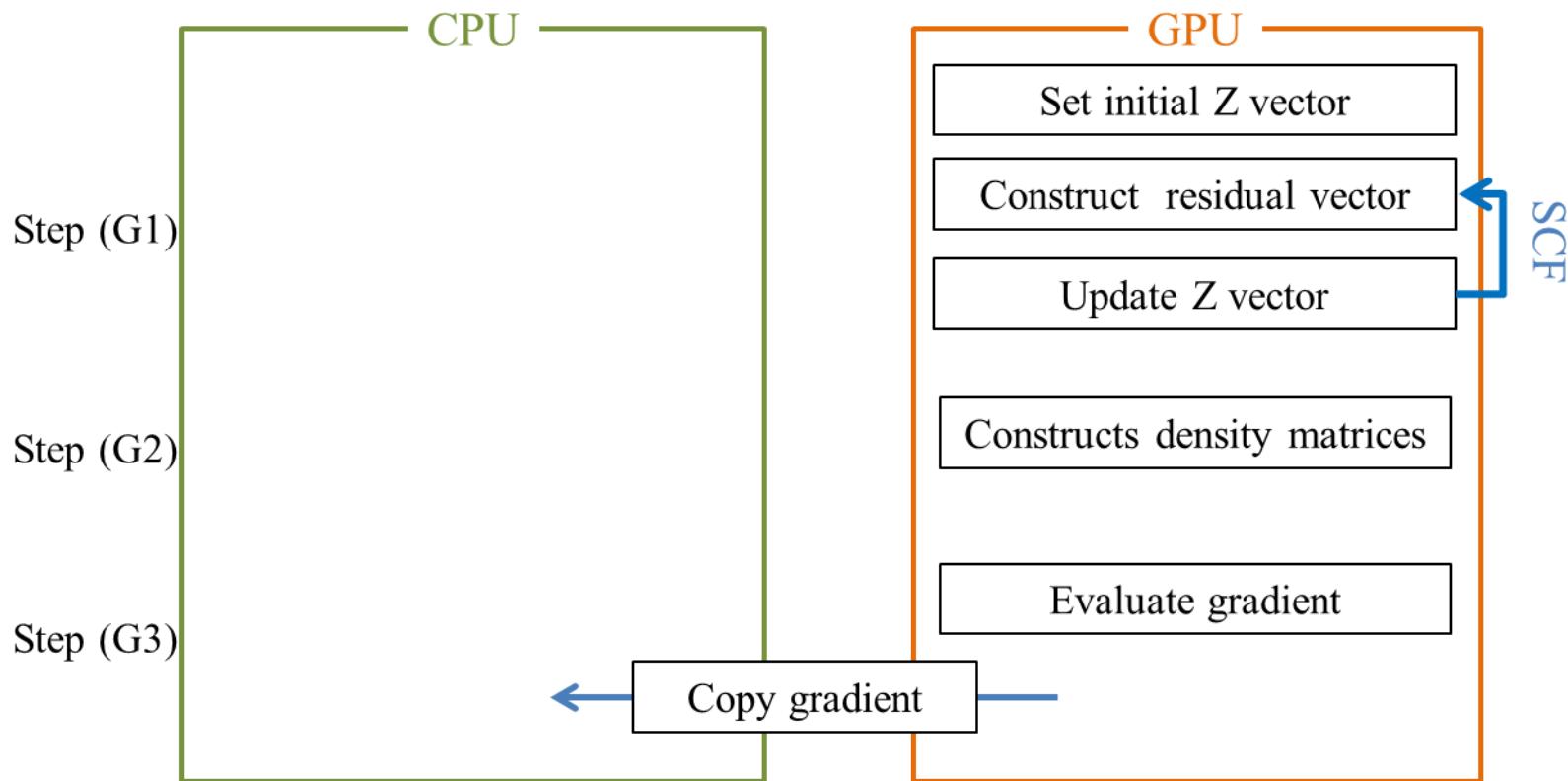
Implementation with GPU: TDDFT energy



✓ In-core algorithm because of implementation with dgemm

$$\Omega_{ia,I} = \omega_{ia}^2 F_{ia} + 4 \sum_A q_A^{ia} \sqrt{\omega_{ia}} \left(\sum_B \gamma_{AB} \left(\sum_{jb} \sqrt{\omega_{jb}} q_A^{jb} F_{jb} \right) \right)$$

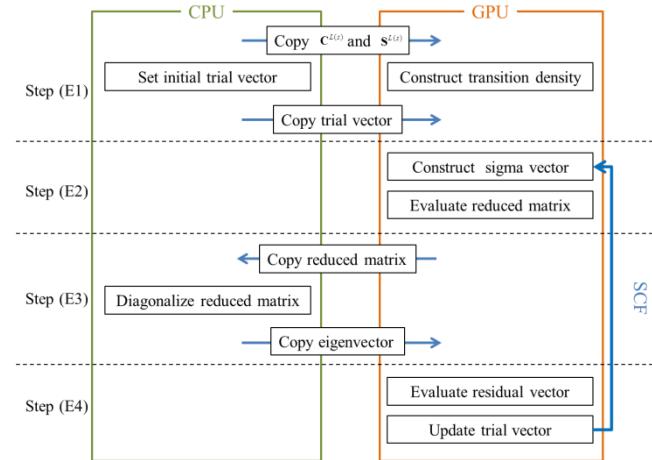
Implementation with GPU: TDDFT energy gradient



- ✓ Acceleration of most procedure with cublas library
CublasDGEMM, CublasDAXPY, Cublasddot
- ✓ DC method: small require memory and communications with CPU and GPU

Performance of (DC-)TDDFTB method

- Wall-clock time [second]:
TDDFTB energy
 - Molecular: *p*-cumaric acid + 228H₂O
 - Method: TDDFTB2/3ob



	TDDFTB CPU	TDDFTB GPU	DC-TDDFTB CPU	DC-TDDFTB GPU
Step 1	0.367	0.013	0.137	0.005
Step 2	1.943	0.075	0.293	0.010
Step 3	0.000	0.002	0.000	0.002
Step 4	0.127	0.003	0.037	0.002

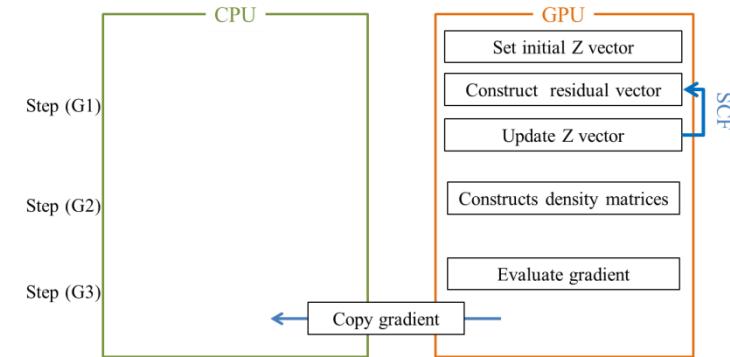
✓ Step 1, 2, and 4: computational cost ↓
Step 3: computational cost ↑

CPU: Intel Xeon Gold 6136 (3.00 GHz) 12 cores, GPU: V100

Performance of (DC-)TDDFTB method

- Wall-clock time [second]:
TDDFTB energy gradient

- Molecular: *p*-cumaric acid + 228H₂O
- Method: TDDFTB2/3ob



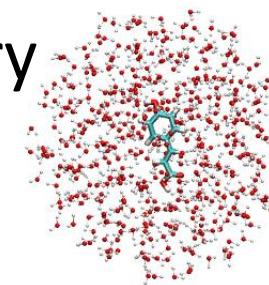
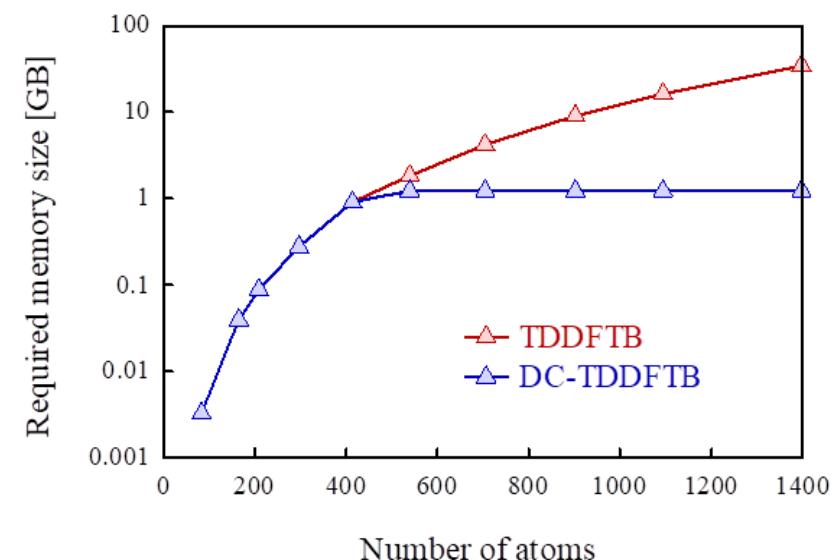
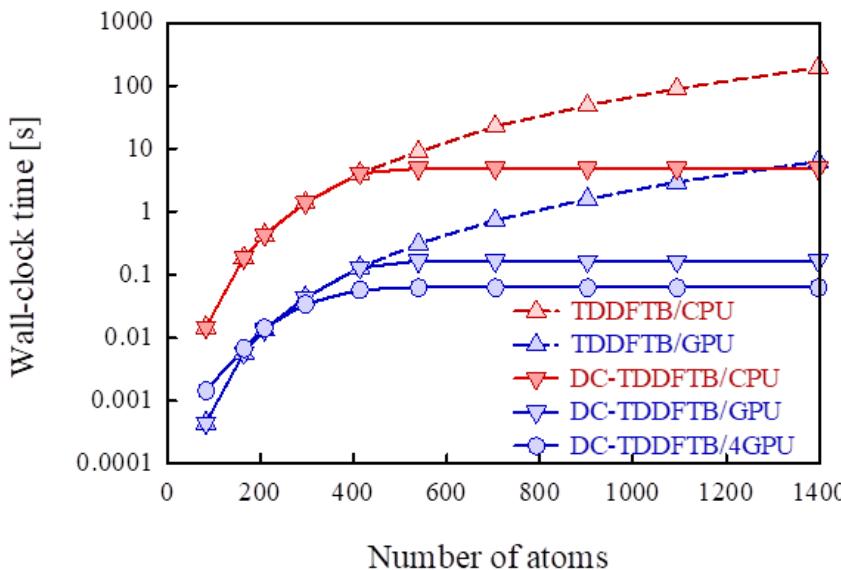
	TDDFTB CPU	TDDFTB GPU	DC-TDDFTB CPU	DC-TDDFTB GPU
Step 1	19.565	0.630	4.222	0.066
Step 2	0.200	0.023	0.066	0.009
Step 3	0.240	0.014	0.091	0.005

✓ All steps: computational cost ↓

Step 4: Required communication cost↑ but small especially DC

System-size dependency: total TDDFT calculation

- Wall-clock time and required memory
 - Molecular: *p*-cumaric acid + $n\text{H}_2\text{O}$
 - Method: TDDFTB2/3ob

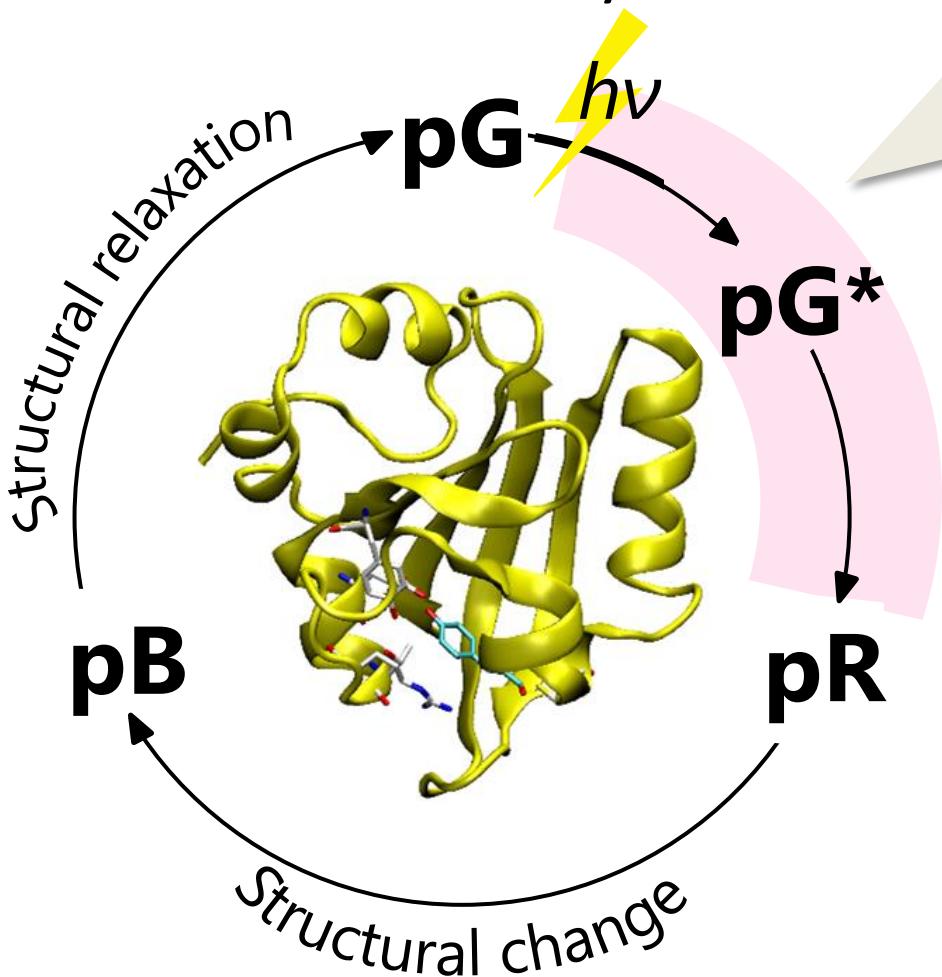


- ✓ DC method: required memory small GPU good!!
- ✓ Multi-GPU: about 3.12 times with 4 GPUs on same board

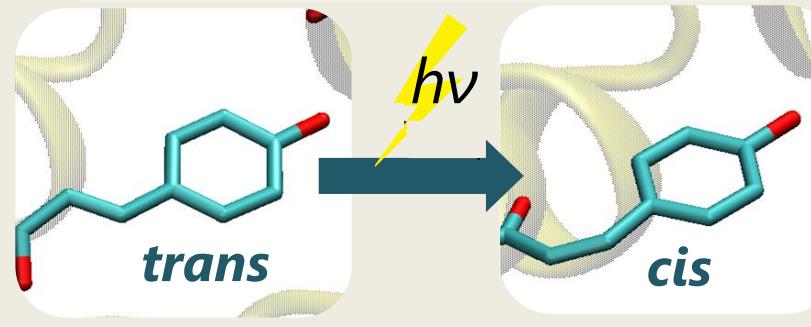
CPU: Intel Xeon Gold 6136 (3.00 GHz) 12 cores, GPU: V100

Application: photoactive yellow protein

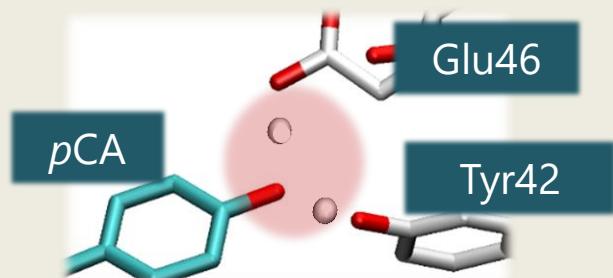
- Photoreaction cycle



Photoisomerization
p-cumaric acid(*p*CA)

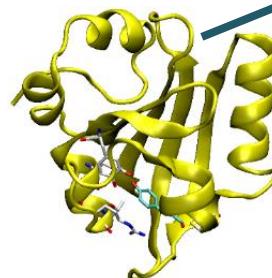
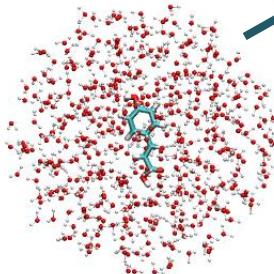


Hydrogen bond
for around amino acid
In water In PYP
284 nm 446 nm
redshift



Application: photoactive yellow protein

Method	Excitation energy [eV]				CPU time
	<i>p</i> CA+461H ₂ O	(diff.)	PYP	(diff.)	
DC-CIS	4.64	(0.64)	4.22	(1.44)	~30 min.
DC-TDBLYP	3.76	(-0.24)	2.33	(-0.45)	~3 hours
DC-TDLCBLYP	4.10	(-0.27)	2.85	(0.07)	~3 hours
DC-TDDFTB	3.68	(-0.32)	2.17	(-0.61)	
DC-TLCDFTB	4.07	(-0.30)	2.80	(0.02)	
Exptl.	4.00	—	2.78	—	



TDBLYP: 6-31G**,
TDDFTB/3ob, Excited center: *p*
1 amino acid, *p*CA, Buffer size:
Å (for PYP),
16 cores of an Intel Xeon Gold

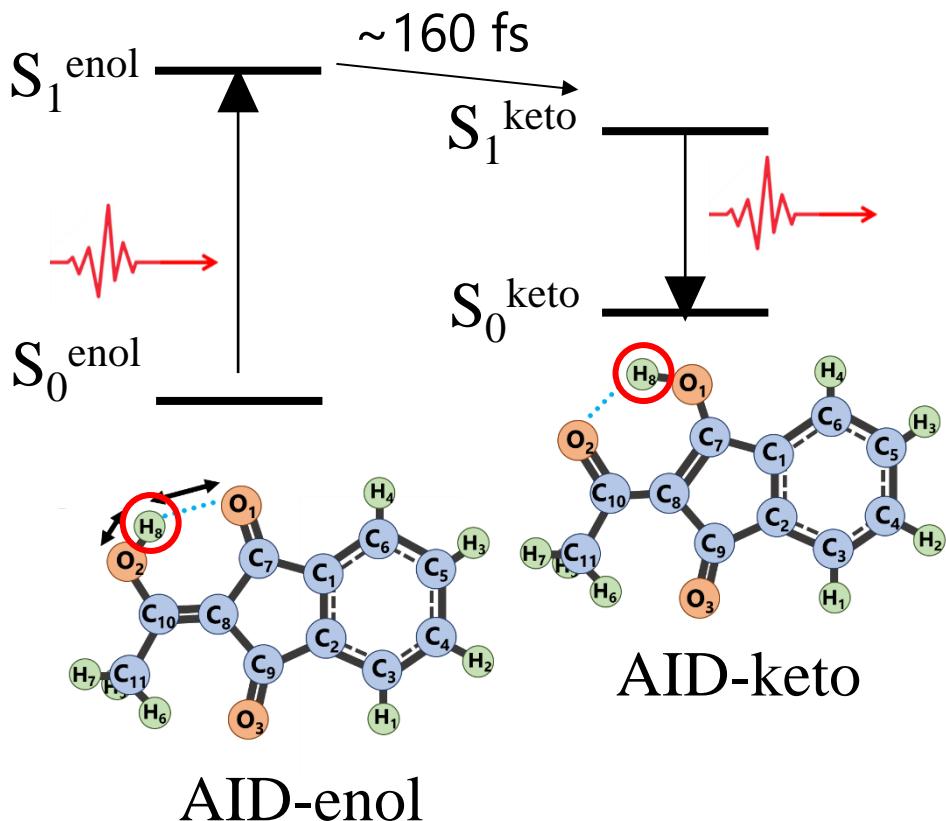


- ✓ TDDFTB method could reproduce the excitation energy of TDDFT calculations within seconds in single point

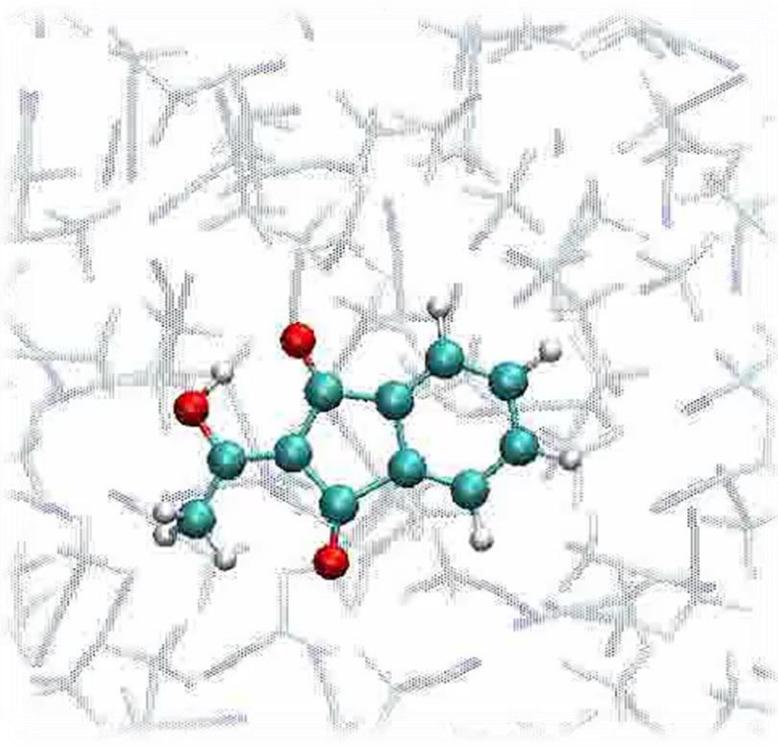
JCTC, 15, 2019
(front cover)

Application: excited-state single proton transfer

Excited state proton transfer
of 2-acetylindan-1,3-dione (AID)



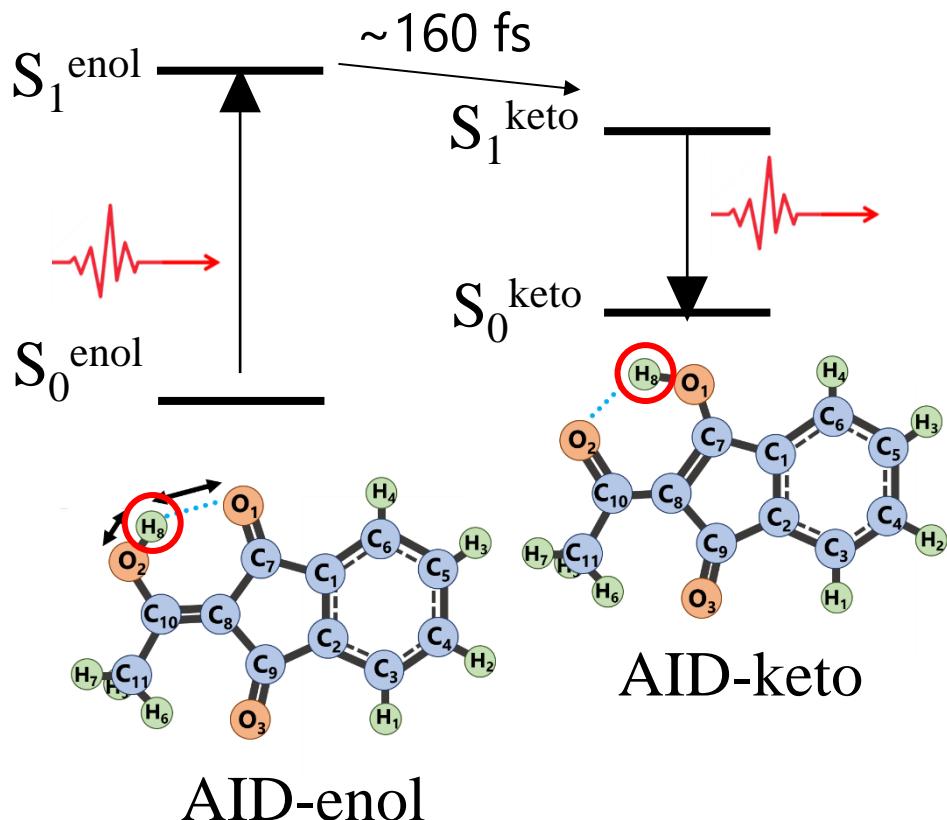
Ground state



- 1 ps DC-DFTB-MD simulation in ground state
- Solvent: acetonitrile
- NVE ensembles
- 3ob parameter set
- Buffer size: 5.0 Å

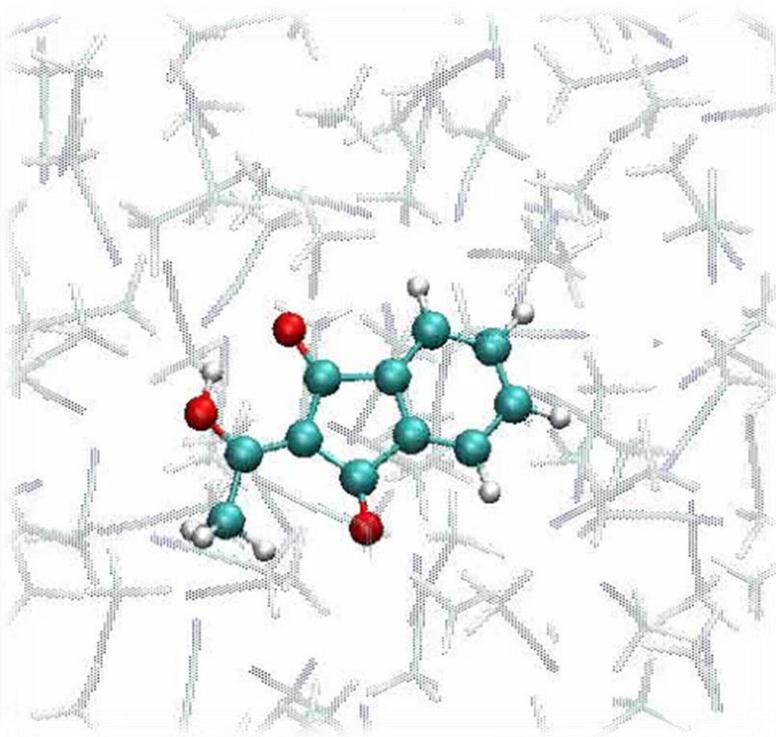
Application: excited-state single proton transfer

Excited state proton transfer
of 2-acetylindan-1,3-dione (AID)



1 ps simulations
CPU: ~11 hours GPU: ~20 mins

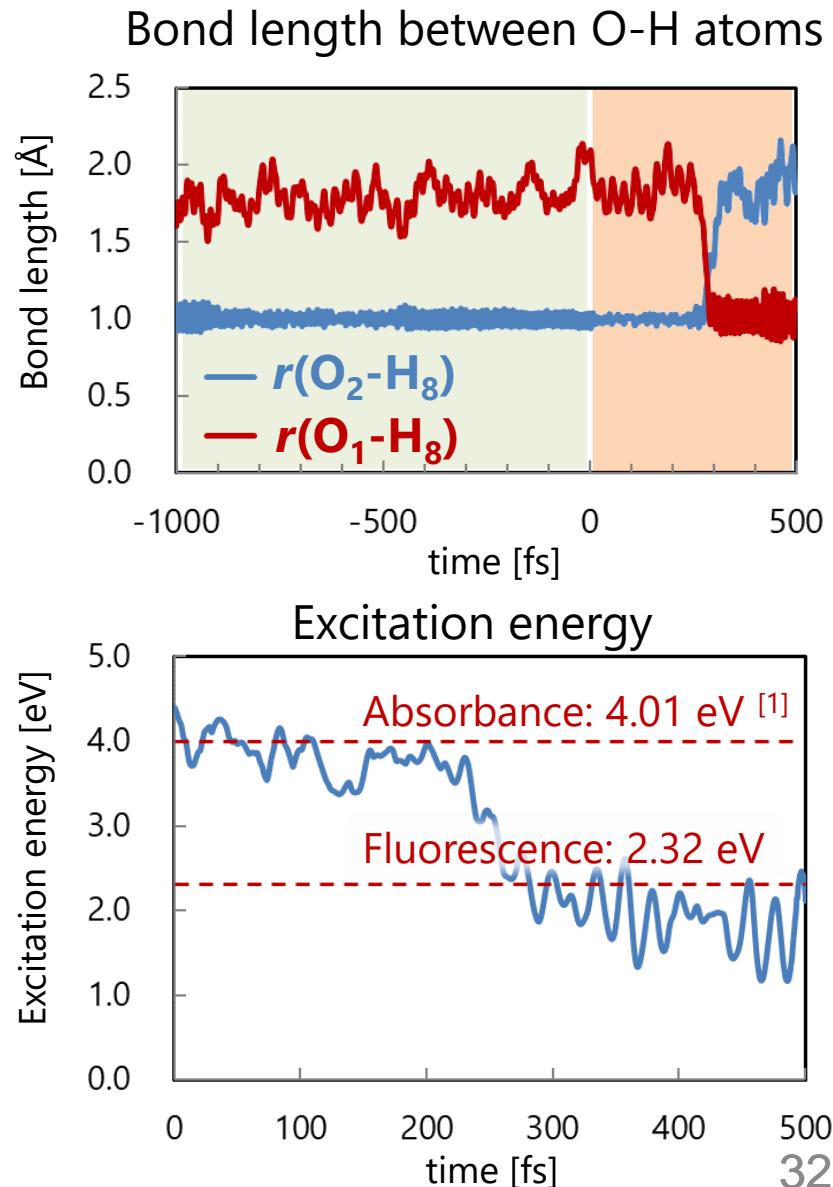
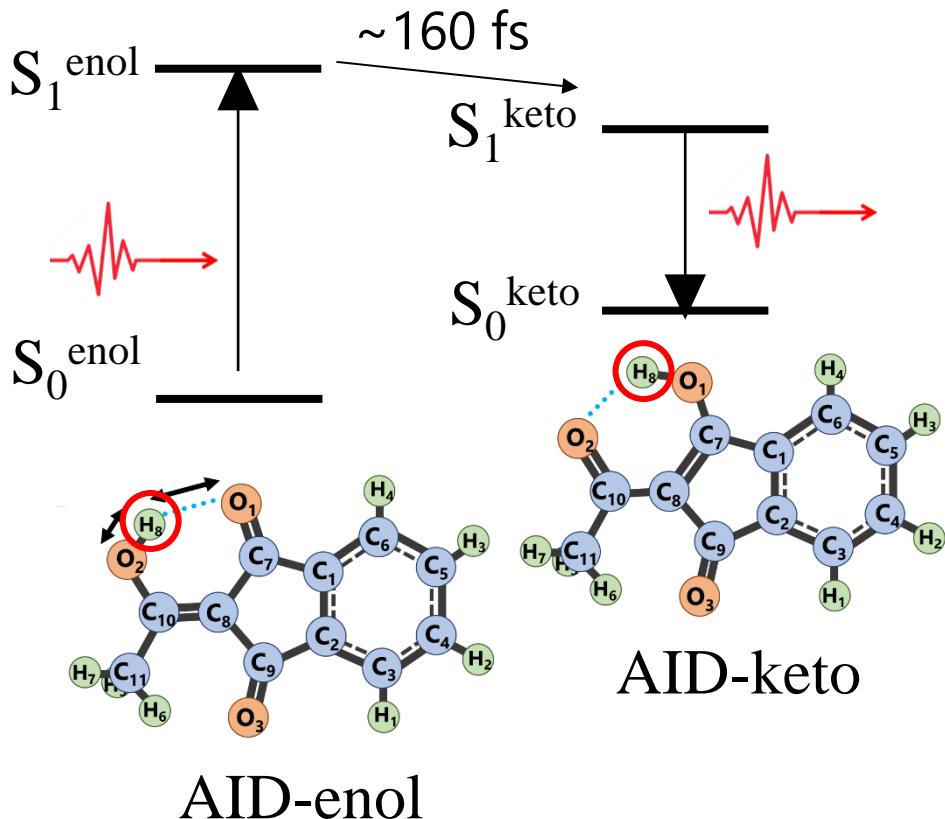
Excited state



- 1 ps DC-DFTB-MD simulation in ground state
- Solvent: acetonitrile
- NVE ensembles
- 3ob parameter set
- Buffer size: 5.0 Å

Application: excited-state single proton transfer

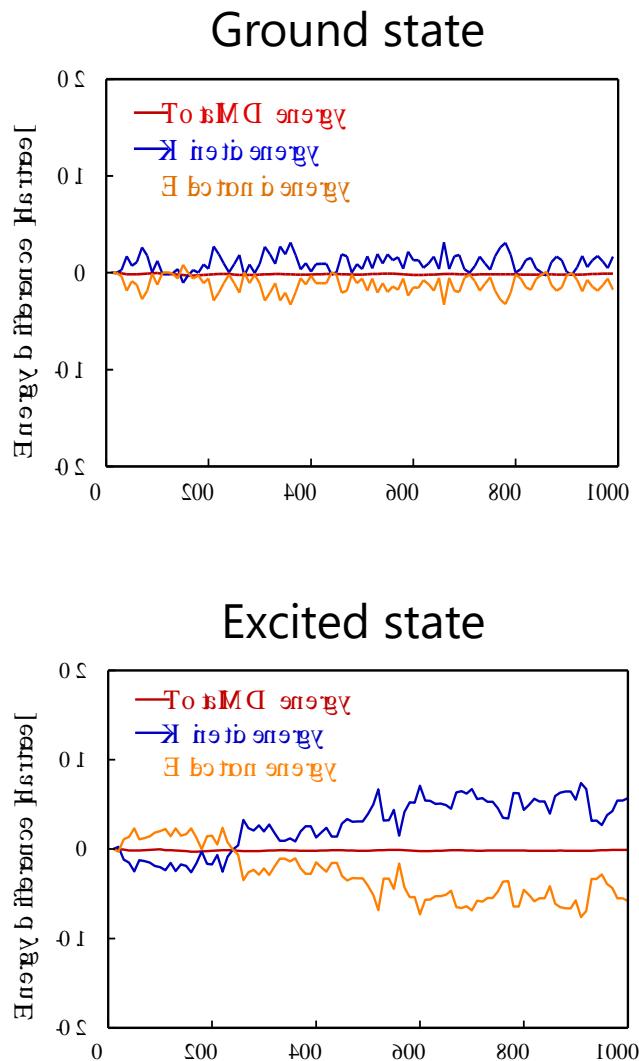
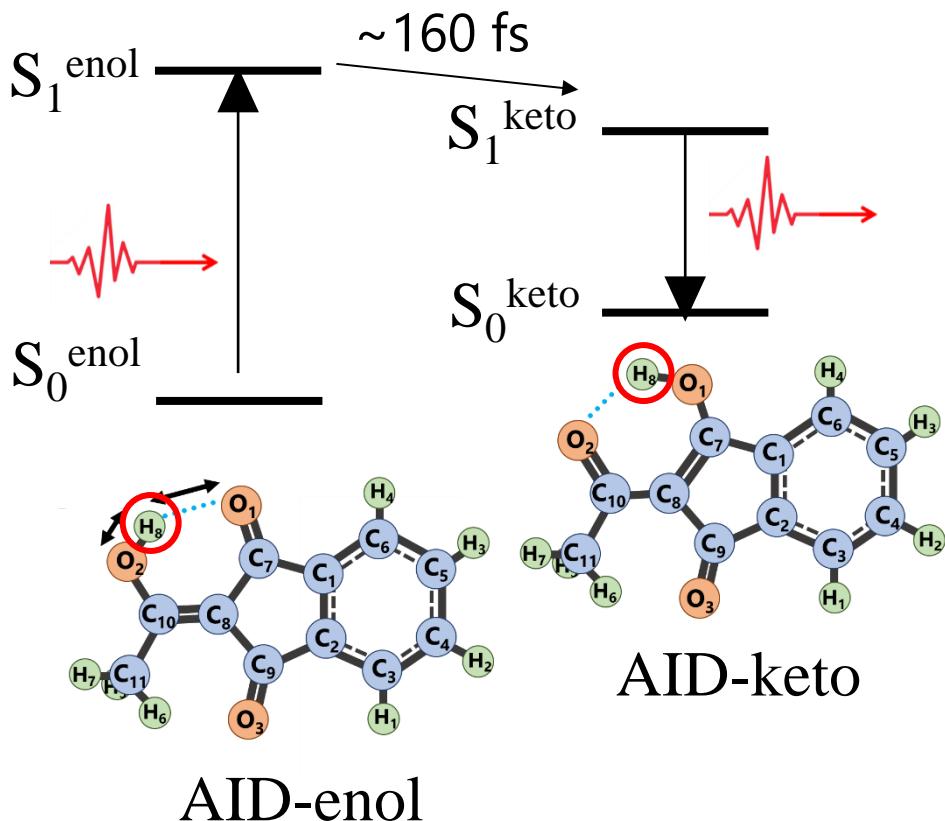
Excited state proton transfer
of 2-acetylindan-1,3-dione (AID)



[1] P. K. Verma, A. Steinbacher, A. Schmiedel, P. Nuernberger, T. Brixner, *Struct. Dyn.*, **3**, 023606 (2015).

Application: excited-state single proton transfer

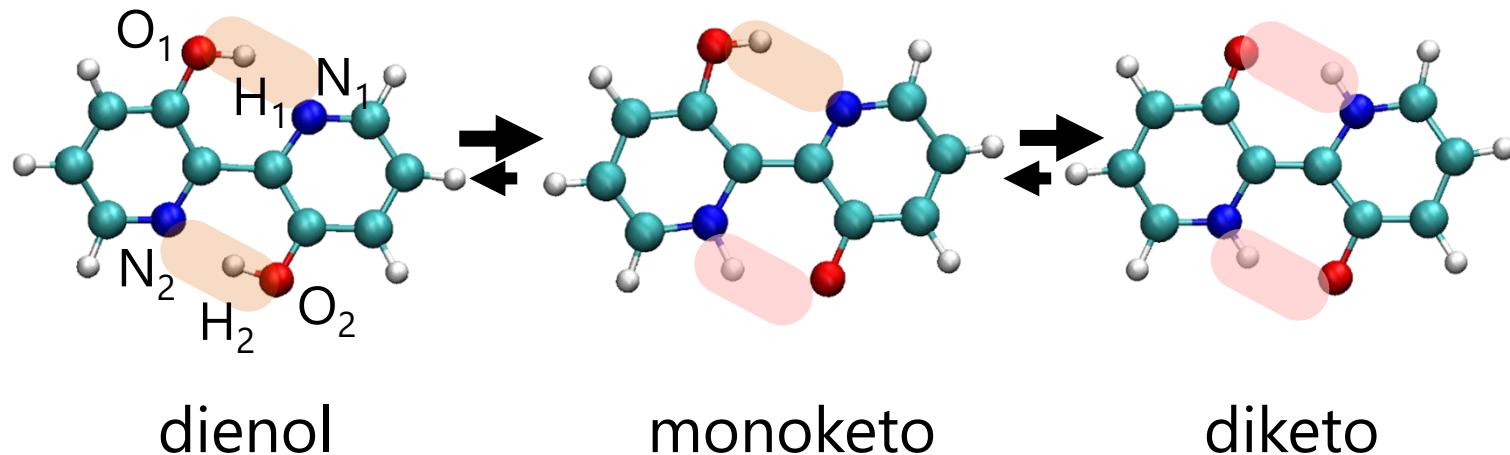
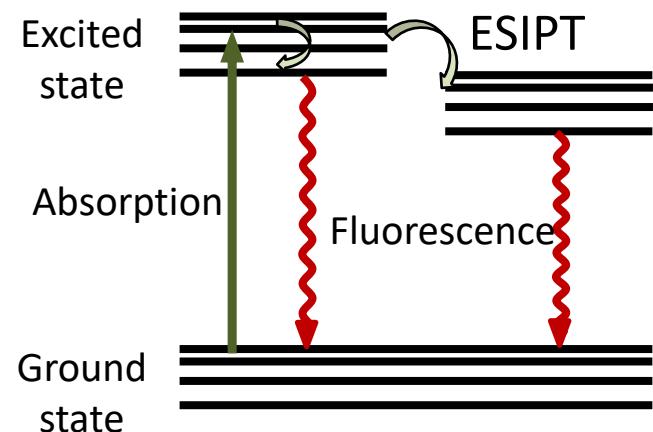
Excited state proton transfer
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Excited-state intramolecular proton transfer (ESIPT)

- Application for organic electro-luminescence and biosensor
- Large red-shift fluorescence spectrum
- 2,2'-Bipyridine-3,3'-diol (BP(OH)_2)
 - Excited-state intramolecular double proton transfer in polar solvent

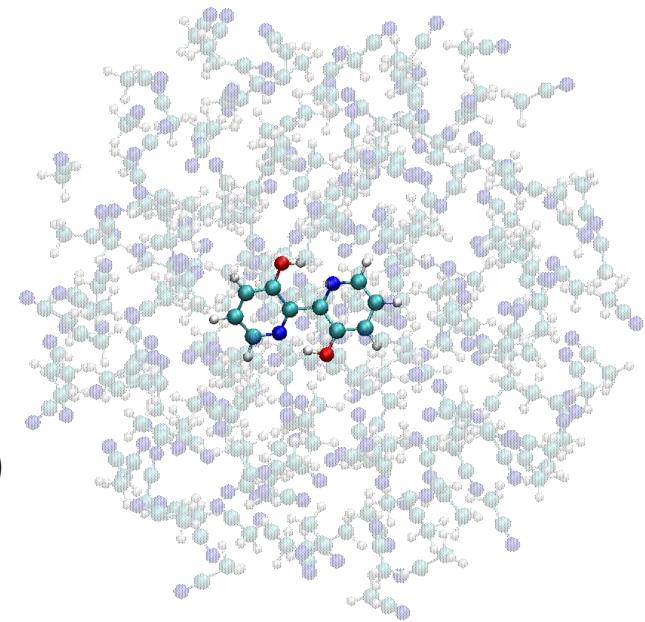


Excited-state intramolecular proton transfer (ESIPT)

- Computational details

Model

$\text{BP(OH)}_2 + 185$ acetonitrile molecules (droplet) after equilibration



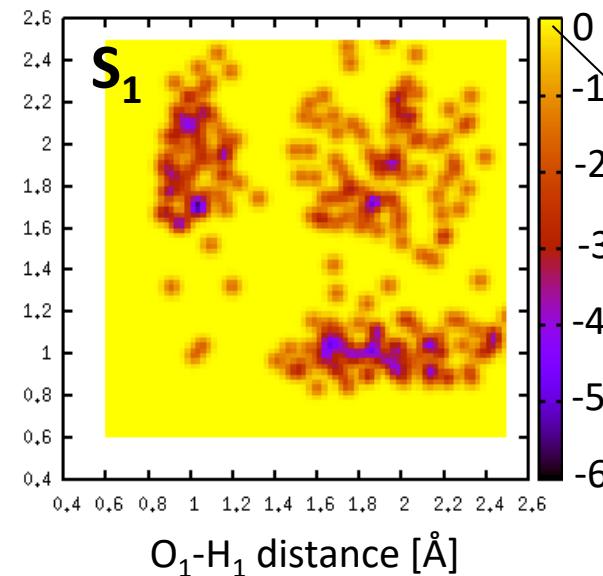
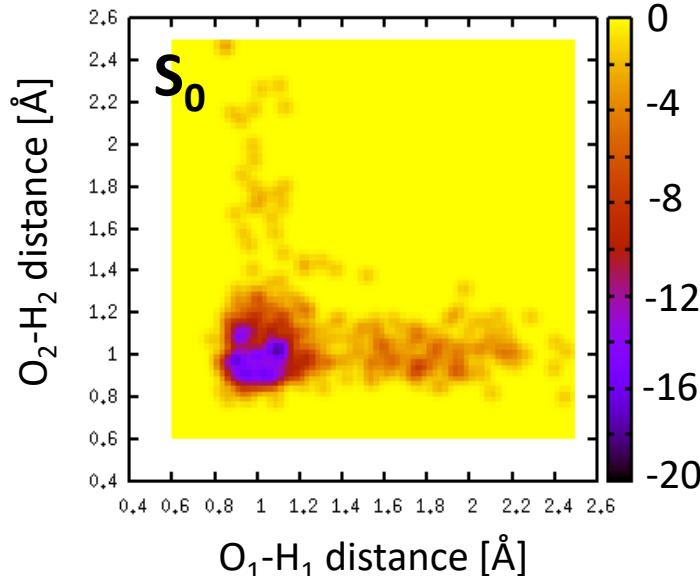
DC-(TD)LCDFTB metaDynamics

- ✓ NVT run for 9 ps at 300 K in ground state (DC-LCDFTB)
- ✓ NVE run for 5 ps in excited state (DC-TDLCDFTB)
- ✓ $\Delta t = 0.5$ fs
- ✓ Parameter: OB2^[1]
- ✓ Subsystem: BP(OH)_2 , acetonitrile
- ✓ Excitation center: BP(OH)_2
- ✓ Buffer size: 5.0 Å
- ✓ Soft potential: $r = 17.0$ Å
- ✓ Dispersion correction: Grimme's DFT-D3
- ✓ Corrective variable: O-H distance
- ✓ Gaussian potential height: 1.88 kcal·mol⁻¹
- ✓ Gaussian potential width: 0.025 Å
- ✓ Frequency of gaussian potential addition : 20 fs
- ✓ Bias temperature: 9000 K

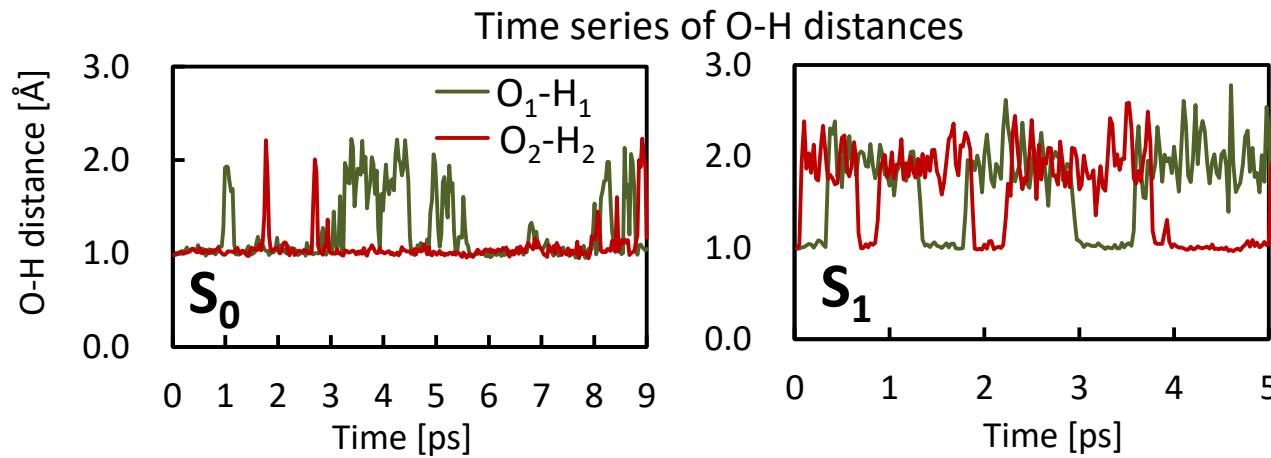
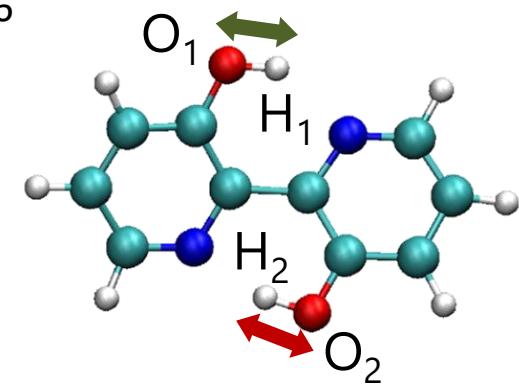
[1] V. Q. Vuong, J. A. Kuriappan, M. Kubillus, J. J. Kranz, T. Mast, T. A. Niehaus, S. Irle, and M. Elstner, *J. Chem. Theory Comput.*, **14**, 115 (2018)

Excited-state intramolecular proton transfer (ESIPT)

- Free energy surface for O-H distances



Free energy difference [kcal · mol⁻¹]



✓ Proton transfer occurred easily in S_1 state.

Summary

- Development of Divide-and-conquer-based time-dependent density-functional tight-binding theory for large excited-state systems: DC-TDDFTB-MD
- Compatibility of DC and GPU
- Further work: Ground state for GPU more large and long-time simulation

